Summer Fields School

KAILASH COLONY, NEW DELHI-110048

Roll No.										

A

- Please check that this questionnaire contains **15** printed pages.
- Code A, B or C given on the right hand top corner of the questionnaire should be written on the answer sheet in the space provided.
- Please check that this questionnaire contains **60** questions.

37TH ARYABHATTA INTER-SCHOOL MATHEMATICS COMPETITION – 2020

CLASS - XI

Time Allowed: **2 Hours** Max. Marks: **100**

GENERAL INSTRUCTIONS:

1. Do not write your name on the questionnaire.
2. Write your roll no. on the questionnaire and the Answer Sheet in the space provided.
3. All the questions are compulsory.
4. Read questions carefully; think twice before you write the answer. **No overwriting or cutting is allowed on the Answer Sheet.** Another copy of the questionnaire or answer sheet will not be provided.
5. Do your rough work in the space provided in the questionnaire.
6. The questionnaire contains three sections. **Section A** contains 30 Multiple Choice Questions of 1mark each, **Section B** contains 20 Free Response Type Questions of 2 marks each and **Section C** contains 10 Free Response Type Questions of 3 marks each.
7. No working or descriptive answers of any question is to be given. Only the Answers are to be written on the Separate Answer sheet provided to you.
8. Use Blue or Black pens to write the answer on the Answer Sheet.
9. Answers should be written clearly in the space provided on the Answer sheet.
10. Use of calculator is not allowed.

SECTION-A

Write the correct option (A, B, C or D) in the Answer sheet.

1. The largest term common to the sequences 1, 11, 21, 31, . . . to100 terms and 31, 36, 41, 46, . . . to 100 terms is
   (A) 381 (B) 471 (C) 281 (D) None of these

2. The smallest positive integral value of n for which $\left(1+\sqrt{3}\,i\right)^{\frac{n}{2}}$ is real is
   (A) 3 (B) 6 (C) 0 (D) 12

3. The number of irrational terms in the expansion of $\left(\sqrt[8]{5}+\sqrt[6]{2}\right)^{100}$ is
   (A) 97 (B) 98 (C) 96 (D) 99

4. The minimum value of $4^x+4^{1-x}, x\in$ R, is
   (A) 2 (B) 4 (C) 1 (D) None of these

5. The value of $\left(i^{-25}+i^{18}\right)$ is
   (A) $-1+i$ (B) $-1-i$ (C) $2+2i$ (D) None of these

SPACE FOR THE ROUGH WORK

6. Let $z=\frac{\cos\theta+i\sin\theta}{\cos\theta-i\sin\theta},\frac{\pi}{4}<\theta<\frac{\pi}{2}$. then arg z is

(A) 2θ (B) $2\theta-\pi$ (C) $\pi+2\theta$ (D) None of these

7. If $\sin\theta+\cos ec\theta=2$ then the value of $\sin^8\theta+\cos ec\theta^8\theta$ is equal to

(A) 2 (B) 2^8 (C) 2^4 (D) None of these

8. The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is atleast 166 cm, then the minimum length of the shortest side (in cm) is

(A) 52 (B) 48 (C) 41 (D) None of these

9. If A=$\left\{x:x\in Z,\ 2^{(x+1)\left(x^2+x-2\right)}=1 \text{ and } -3<x<3\right\}$, then the number of subsets of A is

(A) 32 (B) 16 (C) 8 (D) None of these

10. The equations of the three sides of a triangle are $x=2, y+1=0$ and x+2y=4. The coordinates of the circumcenter of the triangle are

(A) (4, 0) (B) (2, –1) (C) (0, 4) (D) None of these

SPACE FOR THE ROUGH WORK

11. The value of $\sum_{r=1}^{10} r.{}^{r}P_{r}$ is

(A) ${}^{11}P_{11}$ (B) ${}^{11}P_{11}-1$ (C) ${}^{11}P_{11}+1$ (D) None of these

12. In the ellipse $x^2+3y^2=9$, the distance between the foci is

(A) $\sqrt{6}$ (B) 3 (C) $2\sqrt{6}$ (D) None of these

13. Let *x be* the AM and *y, z* be two GMs between two positive numbers, then $\frac{y^3+z^3}{xyz}$ is equal to

(A) 1 (B) 2 (C) 0.5 (D) None of these

14. If $xe^{xy}-y=\sin^2 x$ then $\frac{dy}{dx}$ at $x=0$ is

(A) 0 (B)1 (C) –1 (D) None of these

15. Two cards are drawn at random from a pack of 52 cards. The probability of getting at least a spade and an ace is

(A) $\frac{1}{52}$ (B) $\frac{4}{21}$ (C) $\frac{1}{26}$ (D) None of these

SPACE FOR THE ROUGH WORK

16. Let z be a complex number such that the imaginary part of z is non zero and $a = z^2 + z + 1$ is real, then the value of 'a' cannot be

(A) –1 (B) 1/3 (C) 3/4 (D) None of these

17. The set of real values of x satisfying $\left||x-1|-1\right| \le 1$ is

(A) $[-1,3]$ (B) $[0,2]$ (C) $[-1,1]$ (D) None of these

18. $\lim\limits_{x\to 2} \dfrac{\sqrt{x-2}+\sqrt{x}-\sqrt{2}}{\sqrt{x^2-4}}$ is equal to

(A) 0.5 (B) 1 (C) 2 (D) None of these

19. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are less than 3 then

(A) $a < 2$ (B) $2 \le a \le 3$ (C) $3 < a \le 4$ (D) None of these

20. If the rth term is the middle term in the expansion of $\left(x^2 - \dfrac{1}{2x}\right)^{20}$ then the $(r+3)^{th}$ term is

(A) ${}^{20}C_{14} \cdot \dfrac{1}{2^{14}} . x$ (B) ${}^{20}C_{12} \cdot \dfrac{1}{2^{12}} . x^2$ (C) $-{}^{20}C_{7} \cdot \dfrac{1}{2^{13}} . x$ (D) None of these

SPACE FOR THE ROUGH WORK

21. If the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is e then the eccentricity of the hyperbola $\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$ is

(A) e (B) $\frac{e}{\sqrt{e^2-1}}$ (C) $e\sqrt{e^2-1}$ (D) None of these

22. $\lim_{x\to 0}\frac{x-\tan x}{x\tan x}$ is equal to

(A) 0 (B) 1 (C) 0.5 (D) None of these

23. Two distinct numbers are selected at random from the first 12 natural numbers. The probability that the sum will be divisible by 3 is

(A) $\frac{1}{3}$ (B) $\frac{23}{66}$ (C) $\frac{1}{2}$ (D) None of these

24. The sum of n terms of the series S= $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots$ is equal to

(A) 2^n-n+1 (B) $1-2^{-n}$ (C) $2^{-n}+n-1$ (D) None of these

25. The number of ways in which three letters can be posted in five letter boxes is

(A) 60 (B) 56 (C) 85 (D) None of these

SPACE FOR THE ROUGH WORK

26. If $X=\{4^n-3n-1 : n\in N\}$ and $Y=\{9(n-1) : n\in N\}$ where N is the set of natural numbers then $X\cup Y$ is equal to

(A) X (B) Y–X (C) Y (D) None of these

27. Three identical dice are rolled. The probability that the same number will appear on each of them is

(A) $\frac{1}{6}$ (B) $\frac{1}{36}$ (C) $\frac{1}{18}$ (D) None of these

28. Let T_n be the sum of all possible triangles formed by joining the vertices of an n-sided regular polygon. If $T_{n+1}-T_n=10$, then n is equal to

(A) 5 (B) 10 (C) 8 (D) None of these

29. If the mean deviation about the median of the numbers x, $2x$, $3x$, . . ., $50x$ is 50, then x is equal to

(A) 2 (B) 3 (C) 4 (D) None of these

30. Let A and B be two sets containing 2 and 4 elements respectively. The number of subsets of A×B having 3 or more elements is

(A) –220 (B) 219 (C) 211 (D) None of these

SPACE FOR THE ROUGH WORK

SECTION-B

Write the Answers only in the space provided on the Answer sheet.

31. Find the equation of the straight line which passes through the point (–4, 3) such that the portion of the line between the axes is divided internally by the point in the ratio 5 : 3.

32. Find the polar form of the complex number $z = (1+7i)(2-i)^{-2}$.

33. Find the most general solution of the equation $\sec^2 x = \sqrt{2}\left(1-\tan^2 x\right)$.

34. If $a\,f(x+1)+b\,f\left(\frac{1}{x+1}\right) = x, x \neq -1, a \neq b$, then find $f(2)$.

SPACE FOR THE ROUGH WORK

35. Find the solution set for the following system of inequalities: $\frac{2x+1}{7x-1}>5,\ \frac{x+7}{x-8}>2$

36. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English Dictionary. Find the number of words that appear before the word COCHIN.

37. In a $\triangle ABC$, the sides a, b and c are such that they are the roots of $x^3-11x^2+38x-40=0$. Then find the value of $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}$.

38. Find the largest set of real values of x for which $f(x)=\sqrt{(x+2)(5-x)}-\frac{1}{\sqrt{x^2-4}}$ is a real function.

SPACE FOR THE ROUGH WORK

39. Find the number of words that can be formed from the letters of the word 'PERMUTATIONS' if there are always 4 letters between P and T. (Write the answer in whole number)

40. Find the value of $\sqrt{-3-4i}+\sqrt{3+4i}$ by taking the value of a square root with positive real part only.

41. Find the sum of all the numbers of four different digits that can be formed by the digits 0, 1, 2 and 3.

42. Find the term independent of x in the expansion of $(1-x)^2.\left(x+\frac{1}{x}\right)^{10}$. (Write the answer in whole number)

SPACE FOR THE ROUGH WORK

43. Find the value of $\sin 78^{\circ} - \sin 66^{\circ} - \sin 42^{\circ} + \sin 6^{\circ}$.

44. If z is a complex number satisfying the relation $|z+1| = z + 2(1+i)$, find z.

45. Let $f(\theta) = \dfrac{\cot\theta}{1+\cot\theta}$ and $\alpha+\beta = \dfrac{5\pi}{4}$, then find the value of $f(\alpha).f(\beta)$.

46. Find the number of arrangements of the letters of the word MATHEMATICS taken four letters at a time.

SPACE FOR THE ROUGH WORK

47. If $\sin\alpha, 1$ and $\cos 2\alpha$ are in GP, find the general value of α.

48. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Find the equation of the circle with AB as the diameter.

49. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, find $\frac{dy}{dx}$.

50. A circular ring of radius 3 cm is suspended horizontally from a point 4cm vertically above the centre by 4 strings attached at equal intervals to its circumference. If the angle is between two consecutive string is α , then find cos α .

SPACE FOR THE ROUGH WORK

SECTION-C

Write the Answers only in the space provided on the Answer sheet.

51. Find the value of n if ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 11:6:3$.
52. If the mid points of the sides of triangle are (1, 4), (3, 2) and (–1, 3), then find the sum of the lengths of the three medians.
53. A bag contains 3 black, 4 white and 2 red balls, all the balls being different. Find the number of selections of atmost 6 balls containing balls of all the colours.
54. The equations of the sides AB, BC and CA of the $\triangle ABC$ are $y - x = 2, x + 2y = 1$ and $3x + y + 5 = 0$ respectively. Find the equation of the altitude through B.

SPACE FOR THE ROUGH WORK

55. Find the coefficient of x^{11} in the expansion of $\left(1+x^2\right)^4\left(1+x^3\right)^7\left(1+x^4\right)^{12}$.

56. Find the value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$.

57. Find the sum of *n* terms of the series: $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ \ldots$

SPACE FOR THE ROUGH WORK

58. The sum of the coefficients of first three terms in $\left(x-\frac{3}{x^2}\right)^m$, $x \neq 0$, m being natural number is 559. Find the value of m.

59. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later on it was found that three observations 21, 21 and 18 were incorrect. Find the standard deviation (correct upto 2 places of decimal) if the incorrect observations are omitted.

60. If $2+\sqrt{3}\,i$ is a root of the equation $x^2 + px + q = 0$, where $p, q \in \mathrm{R}$, find $p + q$.

SPACE FOR THE ROUGH WORK