Summer Fields School

KAILASH COLONY, NEW DELHI-110048

Roll No.										

A

- Please check that this questionnaire contains **15** printed pages.
- Code A, B or C given on the right hand top corner of the questionnaire should be written on the answer sheet in the space provided.
- Please check that this questionnaire contains **60** questions.

34TH ARYABHATTA INTER-SCHOOL MATHEMATICS COMPETITION – 2017

CLASS - XI

Time Allowed: **2 Hours** Max. Marks: **100**

GENERAL INSTRUCTIONS:

1. Do not write your name on the questionnaire.
2. Write your roll no. on the questionnaire and the Answer Sheet in the space provided.
3. All the questions are compulsory.
4. Read questions carefully; think twice before you write the answer. **No overwriting or cutting is allowed on the Answer Sheet.** Another copy of the questionnaire or answer sheet will not be provided.
5. Do your rough work in the space provided in the questionnaire.
6. The questionnaire contains three sections. **Section A** contains 30 Multiple Choice Questions of 1mark each, **Section B** contains 20 Free Response Type Questions of 2 marks each and **Section C** contains 10 Free Response Type Questions of 3 marks each.
7. No working or descriptive answers of any question is to be given. Only the Answers are to be written on the Separate Answer sheet provided to you.
8. Use Blue or Black pens to write the answer on the Answer Sheet.
9. Answers should be written clearly in the space provided on the Answer sheet.
10. Use of calculator is not allowed.

SECTION-A

Write the correct option (A, B, C or D) in the Answer sheet.

1. If α and β are the roots of the quadratic equation $x^2 - 5x + 1 = 0$, then find the value of $\frac{\alpha^{1001} + \beta^{1001} + \alpha^{1003} + \beta^{1003}}{\alpha^{1002} + \beta^{1002}}$.

   (A) 3 (B) 5 (C) –3 (D) 2

2. If $x = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, find $x^3 + y^3$.

   (A) 970 (B) 810 (C) 0 (D) 360

3. If α and β are the roots of $x^2 + x + 2 = 0$, γ and δ are the roots of $x^2 + 3x + 4 = 0$ then find the value of $(\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta)$.

   (A) 12 (B) 48 (C) 44 (D) None of these

4. In triangle ABC, M is a point on AC such that AM : MC = 3 : 4. N is a point on AB such that CN intersects BM at P and MP : BP=2 : 3. If BN = 12cm, Find AB (in cm).

   (A) 14 (B) 26 (C) 24 (D) None of these

5. From a circle of radius 10cm, a sector subtending an angle of 120^o at the center of the circle was removed. Two cones are formed by joining the straight edges of each, of the sector and the remaining portion of the circle. Find the ratio of the volume of the bigger cone to the smaller one.

   (A) $\sqrt{10}:1$ (B) $1:2$ (C) $\sqrt{10}:\sqrt{6}$ (D) None of these

SPACE FOR THE ROUGH WORK

6. If the sum of the roots of the equation $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{c}$ is zero, find the product of the roots.

(A) $\frac{1}{2}\left(a^2-b^2\right)$ (B) $-\frac{1}{2}\left(\frac{a+b}{c}\right)$ (C) $-\frac{1}{2}\left(a^2+b^2\right)$ (D) None of these

7. Find the value of a so that $x^2-11x+a=0$ and $x^2-14x+2a=0$ may have a root common.

(A) 24 (B) 12 (C) –12 (D) None of these

8. Which term of the AP 51, 47, 43 . . . is a cube of itself?

(A) 11 (B) 14 (C) 12 (D) 13

9. A cuboid has total surface area of 96 cm^2 and the sum of squares of its sides is 48 cm^2. Find the height of the cuboid (in cm).

(A) 6 (B) 2 (C) 8 (D) 4

10. The legs of a right triangle are 16 cm and 8 cm. Find the perimeter (in cm) of the largest square that can be inscribed in the triangle.

(A) $\frac{16}{3}$ (B) $\frac{32}{3}$ (C) $\frac{64}{3}$ (D) 16

SPACE FOR THE ROUGH WORK

11. A man purchased a certain number of pens for ₹180. He kept one for his own use and sold the remaining pens for one rupee more than the cost price of each pen. Besides getting his own pen for nothing, he made a profit of ₹10. Find the number of pens.
(A) 10 (B) 20 (C) 32 (D) 24

12. Find the value of $\sqrt{8+2\sqrt{8+2\sqrt{8+...}}}$.
(A) 12 (B) 5.6 (C) 4 (D) 4.8

13. If $x\cos^3\theta + y\sin^3\theta = \sin\theta\cos\theta$ and $x\cos\theta = y\sin\theta$, evaluate $x^2 + y^2$.
(A) 3 (B) 4 (C) 2 (D) 1

14. Find the remainder when x^8 is divided by $x^2 - 3x + 2$.
(A) 255x–254 (B) $x - 4$ (C)202x + 1 (D) None of these

15. Pranshi travels 600 km to her home partly by train and partly by car. She takes 8 hours if she travels 120 km by train and rest by car. She takes 20 minutes longer if she travels 200 km by train and rest by car. Find the speed of the car in kmph.
(A) 60 (B) 80 (C) 70 (D) 100

SPACE FOR THE ROUGH WORK

16. At what percentage above the CP must an article be marked so as to gain 33% after allowing a discount of 5%.

(A) 30 (B) 50 (C) 40 (D) 45

17. If $x = \frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$.

(A) 3 (B) 4 (C) 6 (D) 1

18. In quadrilateral ABCD, $\angle A + \angle D = 90^o$, then

(A) $AC^2 - BC^2 = AD^2 - BD^2$ (B) $AC^2 - AD^2 = BD^2 - BC^2$

(C) $AC^2 + AD^2 = BD^2 + BC^2$ (D) None of these

19. If $\tan\alpha = \frac{a}{a+1}$ and $\tan\beta = \frac{1}{2a+1}$, find $\alpha+\beta$.

(A) 0 (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

20. ABCD is square of side of length 1 unit. Two sides of the square are tangent to circle and the circle passes through exactly one of the vertices of the square. Find the area of the circle in square units.

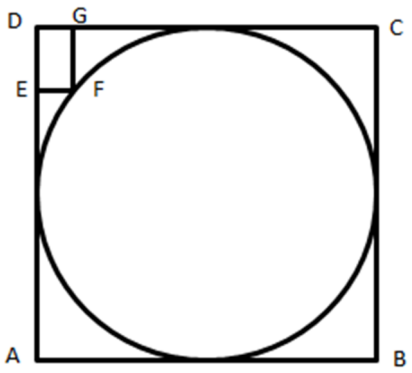
(A) $\left(2-6\sqrt{2}\right)\pi$ (B) $\left(4+6\sqrt{2}\right)\pi$ (C) $\left(4-5\sqrt{2}\right)\pi$ (D) None of these

SPACE FOR THE ROUGH WORK

21. Find the point on x - axis which is equidistant from the points (5, 9) and (–4, 6).

(A) (–2, 0) (B) (3, 0) (C) (2, 0) (D) (–3, 0)

22. In the given figure, ABCD is a square circumscribing a circle. DEFG is a rectangle of sides 20 units and 10 units. Find the circumference of the circle.



(A) 100π units (B) 50π units

(C) 200π units (D) 1000π units

23. The angle of elevation of a cloud from a point 50m above the surface of a lake is 30^o and the angle of depression of its reflection in the lake is 60^o. Find the height of the cloud above the surface of the lake.

(A) 50m (B) 100m (C) 200m (D) None of these

24. Find the area of the triangle ABC with A (1, –4) and the mid points of the sides through A being (0, –1) and (2, –1).

(A) 24 (B) 15 (C) 12 (D) None of these

25. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of the two middle terms is 7 : 15. Find the smallest of the four numbers.

(A) 2 (B) 3 (C) 4 (D) 8

SPACE FOR THE ROUGH WORK

26. The unit digit of the product $14^{124} \times 29^{223}$ is

(A) 8 (B) 2 (C) 6 (D) 4

27. A number when divided by a divisor leaves a remainder of 27. Twice the number divided by the same divisor leaves a remainder of 3. Find the divisor.
(A) 29 (B) 53 (C) 51 (D) None of these

28. Find the value of k for which the following system of equations has infinite solution.
$(k-1)x + ky = k+1, \quad 2kx + (k+6)y = 4k.$
(A) 3 (B) –2 (C) 6 (D) None of these

29. In a circle, A chord AB of length 12 cm is bisected by a diameter CD at P so that CP = 3 cm. Find the diameter of the circle (in cm).
(A) 15 (B) 8 (C) 24 (D) 6

30. Find the distance between the orthocenter and the circumcentre of the triangle whose vertices are (5, 7), (4, 10) and (6, 9).
(A) $\frac{\sqrt{10}}{2}$ (B) 5 (C) $\sqrt{5}$ (D) None of these

SPACE FOR THE ROUGH WORK

SECTION-B

Write the Answers only in the space provided on the Answer sheet.

31. If $z = \sqrt{5+12i} + \sqrt{12i-5}$, find the smallest positive principal value of the argument of z.
32. In a geometric progression consisting of positive terms, each term is equal to the sum of next two terms. Find the common ratio of the GP.
33. Find the constant term in the expansion of $\left(1+x+\frac{2}{x}\right)^6$.
34. Three numbers are chosen from 1 to 30. Find the probability that they are not consecutive.

SPACE FOR THE ROUGH WORK

35. Find the number of all possible words formed by the letters of the word "MATHEMATICS" taken 4 letters at a time.

36. The function f satisfies the functional equation $3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30,$ for all real $x\neq 1.$ Find the value of $f(7)$.

37. Find the coefficient of x^n in the expansion of $(1+x)(1-x)^n$.

38. Find the smallest positive integral value of n for which $\left(\frac{1-i}{1+i}\right)^n$ is purely imaginary with positive imaginary part.

SPACE FOR THE ROUGH WORK

39. A straight line through the point A (3, 4) is such that its intercept between the coordinate axes is bisected at A. Find the equation of the line.

40. Find the number of values of x in $[0, 3\pi]$ such that $2\cos^2 x - 5\sin x + 1 = 0$.

41. Find the point diametrically opposite to the point P (1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$.

42. Evaluate: $\sum_{r=2}^{n} 2^r . {}^nC_r$.

SPACE FOR THE ROUGH WORK

43. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, find $\frac{dy}{dx}$ in terms of x and y only.

44. Out of 800 boys in a school, 224 played cricket, 240 played Hockey and 336 played Basketball. Of the total, 64 played both Basketball and Hockey, 80 played Cricket and Basketball, 40 played Cricket and Hockey and 24 played all the three games. Find the number of boys who played exactly two games or did not play any of the three games.

45. If $z\left(2-2\sqrt{3}i\right)^2=i\left(\sqrt{3}+i\right)^4$ then find $\arg(z)$.

46. If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57:16$, then find the value of n.

SPACE FOR THE ROUGH WORK

47. There are four letters and four addressed envelopes. The letters are put into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one of the letters is put into the right envelope.

48. Write the solution set of the inequation $\frac{x-1}{x+1} \geq 1$ in the interval form.

49. Evaluate: $\lim_{x \to \frac{\pi}{4}} \left[\frac{\sin^3 x - \cos^3 x}{x - \frac{\pi}{4}} \right]$.

50. The eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is the reciprocal of the eccentricity of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, find the length of latus rectum of the hyperbola.

SPACE FOR THE ROUGH WORK

SECTION-C

Write the Answers only in the space provided on the Answer sheet.

51. Find the sum to infinity of the following series:
$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+...$

52. Evaluate: $\cos\frac{\pi}{11}+\cos\frac{3\pi}{11}+\cos\frac{5\pi}{11}+\cos\frac{7\pi}{11}+\cos\frac{9\pi}{11}$.

53. The sides BC, CA and AB of triangle ABC are $x+2y=1$, $3x+y+5=0$ and $x-y+2=0$ respectively. Find the equation of the altitude through B.

SPACE FOR THE ROUGH WORK

54. Find the term independent of x in the expansion of $\left(1+x+x^2\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

55. Find the radius of the circumcircle of the triangle whose vertices are (1, 2), (3, –4) and (5, –6).

56. Find the image of the point (1, –2) when the line $3y = 4x - 5$ is the mirror.

57. Evaluate: $\left({}^8C_0+{}^8C_1\right)+\left({}^8C_1+{}^8C_2\right)+\ldots+\left({}^8C_7+{}^8C_8\right)$.

SPACE FOR THE ROUGH WORK

58. Find the equation of a circle in the simplest form which has the portion of the line $3x + 4y = 14$ intercepted by the lines $x - y = 0$ and $11x - 4y = 0$ as a diameter.
59. All the letters of the word 'MOTHER' are arranged in all possible ways to form words which are to be arranged as in a dictionary. What is the word at 300^{th} position?
60. Find the sum of first 9 terms of the series:

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$$

SPACE FOR THE ROUGH WORK