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ARYABHATTA INTER-SCHOOL MATHS COMPETITION 2006

SUMMER FIELDS SCHOOL (MIDDLE)
CLASS VIII

Time allowed : 2½ hrs. M.M. : 100

GENERAL INSTRUCTIONS:

1. *Participant should not write his/her name on the questionnaire.*
2. *Write your roll no. on each page of the questionnaire.*
3. *All questions are compulsory.*
4. *Read questions carefully, think twice before you write the answer. Another copy of the questionnaire will not be provided.*
5. *Do your rough work on the separate sheet supplied to you and ~~pin~~ up the same with the questionnaire.*
6. *Q. Nos. 1 to ~~3~~ carry ~~10~~ marks each,*
   Q. Nos. 4 to 13 carry 3 marks each and
   Q. Nos. ~~14~~ to 23 carry ~~4~~ marks each.
7. *Answers to Q. Nos. 1, 2 and 3 are to be given in the space provided with the questions.*
8. *Q. Nos. 4 to ~~23~~ are to be answered in the space provided after Q. No. ~~23~~.* 19.
9. *Use of calculator is not allowed.*

PART – A

1. Fill in the blanks:

   (i) The smallest number to be added to 5470, to make it a perfect square is .....

   (ii) If $a \star b = a^{2b} \oplus b^2$, $x \oplus y = (x-y)^2$, then $4 \star 3$ = ...

   (iii) $\sqrt[3]{3^{n+2}} = 9$, then $\sqrt{4^{n-1}}$ = ...

   (iv) The total surface area of a cube is 384 cm², then its volume is .....

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(v) $\frac{2}{3}$ of $\frac{1}{5}$ of the complement of an angle is 8°, then the angle is .....

(vi) A sum of money doubles itself at simple interest in 8 years. The rate of interest is .....

(vii) The missing number in the series, 3, ..., 15,31,63,127 is .....

(viii) If b:a = 2 : 1 and c : b = 3 : 1 , then a + b : b + c is .....

(ix) If $x = 3 + 2y$, then $x^3 + 8y^3 - 18xy =$ .....

(x) 0.5 % of 20 % of 8000 is .....

2. State True or False:

(i) $|-x| = -x$ if $x < 0$.

(ii) 2n, n^2-1 and n^2+1 form pythagorean triplets for all natural numbers n>1.

(iii) The orthocentre of a triangle divides each altitude in the ratio 2:1, then the triangle is an equilateral triangle.

(iv) A circle and a square has same perimeters, then the area of the square is more than the area of the circle.

(v) The diagonals of quadrilateral are equal, then it is always a parallelogram.

(vi) $0.3\overline{54} = \frac{351}{990}$.

(vii) $\sqrt[3]{\sqrt{4096}} = 64$.

(viii) The circumcentre of a right isosceles triangle lies on its hypotenuse.

(ix) The angle subtended by a side of a regular polygon at the centre of its circumcircle is 60°, the polygon is a pentagon.

(x) Angle inscribed in a semicircle is always a right angle.

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3. Tick(√) against the correct answer:

(i) The first non-zero digit from the right in the number 30^{2720} is

(a) 9 (b) 7 (c) 3 (d) 1.

(ii) The ratio of the perimeter of a quadrant of a circle to the circumference of the same circle is circle is

(a) $\frac{4\pi}{\pi+4}$ (b) $\frac{\pi}{\pi+4}$ (c) $\frac{\pi+4}{4\pi}$ (d) none of the above.

(iii) The maximum number of points of intersection of 5 circles is

(a) 36 (b) 20 (c) 6 (d) 14.

(iv) $\sqrt{5+\sqrt{10+\sqrt{x}}} = 3$, then the value of x is

(a) 49 (b) 36 (c) 16 (d) 25.

(v) The sum of the circumference and the radius of a circle is 51 cm . The area of the circle is

(a) 158 cm^2 (b) 154 cm^2 (c) 114 cm^2 (d) 256 cm^2.

(vi) Mean proportional of 4 and 36 is $x^3 + 4$, the x is equal to

(a) 4 (b) 6 (c) 2 (d) 12.

(vii) If a and b are natural number then which of the following statements can be true.

(a) $a^2 = 3b^2$ (b) $a^2 = 5b^2$ (c) $a^2 = 8b^2$ (d) $a^2 = 4b^2$

(viii) The number 54738 x32592 is divisible by 11, the digit in place of x is

(a) 1 (b) 3 (c) 0 (d) 9.

(ix) $\frac{10+\sqrt[3]{8}}{98.4+\sqrt{2.56}}$ is equal to

(a) 0.12 (b) 0.012 (c) 1.2 (d) 11.2.

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(x) A cylinder of height 7 m and radius 2 m is carved out of a cuboid of dimensions 10m x 5m x 2m. The percentage of wood wasted is

(a) 15 % (b) 12 % (c) 24 % (d) none of the above.

PART – B

4. A school collected Rs. 404500 for Kashmir Earth Quake Relief Fund. Each student donated the amount in rupees equal to twice the number of students in the school. Find the number of students in the school.

5. If $x = \sqrt{3+\sqrt{5}}$, then show that $x^4 - 6x^2 + 4 = 0$.

6. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$, then prove that $a = b = c$.

7. Find the value of a and b if $x^2 + 1$ is a factor of $x^4 + x^3 + 8x^2 + ax + b$.

8. Two parallel chords of a circle of radius 20 cm, are of length 32 cm and 24 cm. Find the distance between them, if they lie on the same side of the circle.

9. Two congruent circles with centres A and B intersect at P and Q. □APBQ is a square of side 1 cm. Find the area, common in the two circles.

10. Factorize : $x^2 - (y + \frac{1}{y})x + 1$.

11. Solve the following equation for x :

$$\frac{a-x}{a} + \frac{2a-x}{2a} = \frac{3a-x}{3a}.$$

12. Find the least square number, which is exactly divisible by each of the numbers 8, 12, 15, and 20.

13. If a, b, and c are in continued proportion, then prove that

$$\frac{a^3+b^3+c^3}{a^2b^2c^2} = \frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}.$$

14. A father's present age is four times the present age of the elder son and five times the present age of the younger son. When the age of the elder son will be three times of his present age, then the father's age will be 4 years more than twice the age of the younger son at that time. Find their present ages.

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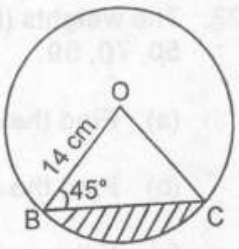
15. An article is sold at 20 % profit. If its cost price is increased by Rs 50 and at the same time if its selling price is also increased by Rs 30, the percentage profit decreases by $3\frac{1}{3}$%. Find the cost price of the article.

16. In $\triangle ABC$, AE bisects $\angle BAC$ and $AD \perp BC$, show that $\angle DAE = \frac{1}{2}(\angle B - \angle C)$.

17. Find the area of the shaded region in the adjacent figure if O is the centre of the circle and OB = 14 cm, $\angle OBC = 45°$.

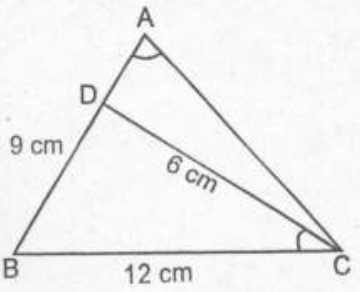


18. Ram Lal and Mohan Lal start working together and complete 60% of the work in 4 days. After that, Ram Lal went to his village to attend a marriage and Mohan Lal completed the work in 8 more days. How long will Ram Lal take to complete that work alone.

19. A sum of money doubles itself at compound interest in 15 years. In how many years will it become 8 times of itself at the same rate of interest, compounded annually.

20. A spherical ball of lead, 3 cm in diameter, is melted and recast into three spherical balls. The diameters of the two of these balls are 1.5 cm. and 2.5 cm respectively. Find the diameter of the third ball.

21. In the adjacent figure BD = 9 cm, BC = 12 cm and CD = 6 cm, and $\angle BAC = \angle BCD$. Find the ratio of the perimeters of $\triangle ADC$ and $\triangle DBC$.



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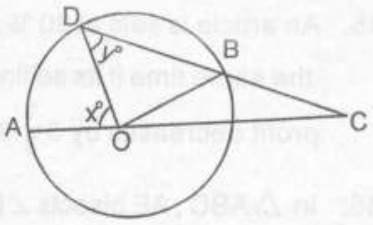
22. In the adjacent figure, O is the centre of the circle and OB = BC, then prove that 2x=3y.



23. The weights (in kg) of 10 students of class are 40, 38, 56, 68, 35, 45, 43, 50, 70, 59.

(a) Find the range of the data.

(b) Find the mean of the data.

(c) If the weight of the class teacher is also included, the mean weight increases by 1.6 kg. Find the weight of the teacher.